Laplace Transforms Revisited

(see FPE, Appendix A)

One-sided (or unilateral) Laplace transform:

$$\mathscr{L}\{f(t)\} \equiv F(s) = \int_0^\infty f(t)e^{-st}\mathrm{d}t \qquad \text{(really, from } 0^-\text{)}$$

— for simple functions f, can compute $\mathscr{L}f$ by hand.

Example: unit step

$$f(t) = 1(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

$$\mathscr{L}\{1(t)\} = \int_0^\infty e^{-st}\mathrm{d}t = -\frac{1}{s}e^{-st}\Big|_0^\infty = \frac{1}{s} \qquad \text{(pole at } s = 0\text{)}$$

— this is valid provided $\mathrm{Re}(s) > 0$, so that $e^{-st} \xrightarrow{t\to+\infty} 0$.