Example (continued)

Compute the step response of

$$\ddot{y} + 3\dot{y} + 2y = u, \qquad y(0) = \alpha, \ \dot{y}(0) = \beta$$

$$Y(s) = \frac{\alpha s^2 + (3\alpha + \beta)s + 1}{s(s+1)(s+2)} \qquad y(t) = \mathscr{L}^{-1}\{Y(s)\}$$

Use the method of partial fractions:

$$\frac{\alpha s^2 + (3\alpha + \beta)s + 1}{s(s+1)(s+2)} = \frac{a}{s} + \frac{b}{s+1} + \frac{c}{s+2}$$

— this gives $a = 1/2, \ b = 2\alpha + \beta - 1, \ c = -\alpha - \beta + 1/2$

$$Y(s) = \frac{1}{2s} + (2\alpha + \beta - 1)\frac{1}{s+1} + \frac{-\alpha - \beta + 1/2}{s+2}$$

$$y(t) = \mathscr{L}^{-1}\{Y(s)\} = \frac{1}{2}1(t) + (2\alpha + \beta - 1)e^{-t} + (1/2 - \alpha - \beta)e^{-2t}$$