Example (continued)

$$\ddot{y} + 3\dot{y} + 2y = u, \qquad y(0) = \alpha,\ \dot{y}(0) = \beta$$

Compute the *step response*, i.e., response to $u(t) = 1(t)$

Caution!! $\quad Y(s) = H(s)U(s)$ no longer holds if $\alpha \neq 0$ or $\beta \neq 0$

Again, take Laplace transforms of both sides, mind the I.C.'s:

$$s^2Y(s) - s\alpha - \beta + 3sY(s) - 3\alpha + 2Y(s) = U(s)$$

$U(s) = \mathscr{L}\{1(t)\} = 1/s$, which gives

$$s^2Y(s) - s\alpha - \beta + 3sY(s) - 3\alpha + 2Y(s) = \frac{1}{s}$$

$$Y(s) = \frac{\alpha s + (3\alpha + \beta) + \frac{1}{s}}{s^2 + 3s + 2} = \frac{\alpha s^2 + (3\alpha + \beta)s + 1}{s(s+1)(s+2)}$$

Note: if $\alpha = \beta = 0$, then $Y(s) = \dfrac{1}{s(s+1)(s+2)} = H(s)U(s)$