Laplace Transforms and Differentiation

Given a differentiable function f, what is the Laplace transform $\mathscr{L}\{f'(t)\}$ of its time derivative?

$$\begin{aligned}\mathscr{L}\{f'(t)\} &= \int_0^\infty f'(t)e^{-st}\mathrm{d}t \\ &= f(t)e^{-st}\Big|_0^\infty + s\int_0^\infty e^{-st}f(t)\mathrm{d}t \qquad \text{(integrate by parts)} \\ &= -f(0) + sF(s)\end{aligned}$$

— provided $f(t)e^{-st} \to 0$ as $t \to \infty$

$\mathscr{L}\{f'(t)\} = sF(s) - f(0)$ — this is how we account for I.C.'s

Similarly:

$$\begin{aligned}\mathscr{L}\{f''(t)\} = \mathscr{L}\{(f'(t))'\} &= s\mathscr{L}\{f'(t)\} - f'(0) \\ &= s^2F(s) - sf(0) - f'(0)\end{aligned}$$