## Transient and Steady-State Response

Consider the system $\dot{y} = -y + u \qquad y(0) = 0$

We computed the response to $u(t) = \cos t$ in two ways:

$$y(t) = -\frac{1}{2}e^{-t} + \frac{1}{\sqrt{2}}\cos(t - \pi/4)$$

— using the method of partial fractions;

$$y(t) = \frac{1}{\sqrt{2}}\cos\left(t - \pi/4\right)$$

— using the frequency response formula.

Q: Which answer is correct? And why?

A: At $t = 0$, $\frac{1}{\sqrt{2}}\cos(t - \pi/4) = \frac{1}{2} \neq 0$, which is inconsistent

with the initial condition $y(0) = 0$. The term $-\frac{1}{2}e^{-t}\big|_{t=0} = -\frac{1}{2}$
cancels the steady-state term, so indeed $y(0) = 0$.

Therefore, the first formula is correct.