

Transient and Steady-State Response

Consider the system $\dot{y} = -y + u$ $y(0) = 0$

$$u(t) = \cos t \quad \longrightarrow \quad y(t) = \underbrace{-\frac{1}{2}e^{-t}}_{\text{transient response}} + \underbrace{\frac{1}{\sqrt{2}} \cos(t - \pi/4)}_{\text{steady-state response}}$$

— transient response vanishes as $t \rightarrow \infty$ (we will see later why)

Let's compare against the frequency response formula:

$$H(s) = \frac{1}{s+1} \quad \Longrightarrow \quad H(j\omega) = \frac{1}{j\omega+1}$$

$u(t) = \cos t$ has $A = 1$ and $\omega = 1$, so

$$\begin{aligned} y(t) &= M(1) \cos(t + \varphi(1)) \\ &= \frac{1}{\sqrt{2}} \cos(t - \pi/4) \end{aligned}$$

— the freq. response formula gives only the steady-state part!!