Laplace Transforms and the Transfer Function

Reminder: the *two-sided* Laplace transform of a function $f(t)$ is

$$F(s) = \int_{-\infty}^{\infty} f(\tau) e^{-s\tau} \mathrm{d}\tau, \qquad s \in \mathbb{C}$$

time domain	frequency domain
$u(t)$	$U(s)$
$h(t)$	$H(s)$
$y(t)$	$Y(s)$

convolution in time domain $\longleftrightarrow$ multiplication in frequency domain

$$y(t) = h(t) \star u(t) \quad \longleftrightarrow \quad Y(s) = H(s)U(s)$$

The Laplace transform of the impulse response

$$H(s) = \int_{-\infty}^{\infty} h(\tau) e^{-s\tau} \mathrm{d}\tau,$$

is called the transfer function of the system.