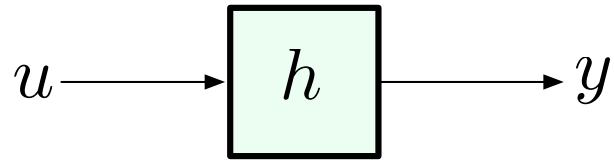
Frequency Response



$$u(t) = A\cos(\omega t) \quad \longrightarrow \quad y(t) = \frac{A}{2}\Big(H(j\omega)e^{j\omega t} + H(-j\omega)e^{-j\omega t}\Big)$$

$$H(j\omega) \in \mathbb{C} \quad \Longrightarrow \quad H(j\omega) = M(\omega)e^{j\varphi(\omega)}$$
$$H(-j\omega) = M(\omega)e^{-j\varphi(\omega)}$$

Therefore,

$$\begin{aligned} y(t) &= \frac{A}{2}M(\omega)\Big[e^{j(\omega t+\varphi(\omega))} + e^{-j(\omega t+\varphi(\omega))}\Big] \\ &= AM(\omega)\cos\big(\omega t + \varphi(\omega)\big) \quad \text{(only true in } \textit{steady state}\text{)} \end{aligned}$$

The (steady-state) response to a cosine signal with amplitude A and frequency ω is still a cosine signal with amplitude $AM(\omega)$, same frequency ω, and phase shift $\varphi(\omega)$