Example

$$\dot{y} = -ay + u \qquad \text{(think } y = x, \text{ full measurement)}$$
$$u(t) = e^{st} \qquad \text{(always assume } u(t) = 0 \text{ for } t < 0)$$
$$y(t) = H(s)e^{st} \qquad \text{— what is } H?$$

Let's use the system model:

$$\dot{y}(t) = \frac{\mathrm{d}}{\mathrm{d}t}\left(H(s)e^{st}\right) = sH(s)e^{st}$$

Substitute into $\dot{y} = -ay + u$:

$$sH(s)e^{st} = -aH(s)e^{st} + e^{st} \qquad (\forall s; t > 0)$$
$$sH(s) = -aH(s) + 1$$

$$H(s) = \frac{1}{s+a} \qquad \Longrightarrow \qquad y(t) = \frac{e^{st}}{s+a}$$