Laplace Transforms and the Transfer Function

$$Y(s) = H(s)U(s), \qquad \text{where } H(s) = \int_{-\infty}^{\infty} h(\tau)e^{-s\tau}\mathrm{d}\tau$$

- So, how should we compute $H(s)$ in practice?

  Try injecting some specific inputs and see what happens at the output.

  Let's try $u(t) = e^{st}, t \geq 0$ (s is some fixed number)

$$\begin{aligned} y(t) &= \int_0^{\infty} h(\tau)u(t-\tau)\mathrm{d}\tau \qquad (\text{because } u \star h = h \star u) \\ &= \int_0^{\infty} h(\tau)e^{s(t-\tau)}\mathrm{d}\tau \\ &= e^{st}\int_0^{\infty} h(\tau)e^{-s\tau}\mathrm{d}\tau \\ &= e^{st}H(s) \end{aligned}$$

  – so, $u(t) = e^{st}$ is multiplied by $H(s)$ to give the output.