Laplace Transforms and the Transfer Function

$$Y(s) = H(s)U(s),$$
 where $H(s) = \int_{-\infty}^{\infty} h(\tau)e^{-s\tau} d\tau$

Given u(t), we can find U(s) using tables of Laplace transforms or MATLAB. But how do we know h(t) [or H(s)]?

Suppose we have a state-space model:

$$u \xrightarrow{\qquad } \begin{array}{c} \dot{x} = Ax + Bu \\ y = Cx \end{array} \xrightarrow{\qquad } y$$

In this case, we have an exact formula:

$$H(s) = C(Is - A)^{-1}B \qquad \text{(matrix inversion)}$$
$$h(t) = Ce^{At}B, \ t \ge 0^{-} \qquad \text{(matrix exponential)}$$

— will not encounter this until much later in the semester.