The Main Result: Separation Principle

So now we can write

$$\begin{pmatrix} \dot{x} \\ \dot{e} \end{pmatrix} = \underbrace{\begin{pmatrix} A - BK & BK \\ 0 & A - LC \end{pmatrix}}_{\text{upper triangular matrix}} \begin{pmatrix} x \\ e \end{pmatrix}$$

The closed-loop characteristic polynomial is

$$\det \begin{pmatrix} Is - A + BK & -BK \\ 0 & Is - A + LC \end{pmatrix}$$
$$= \det (Is - A + BK) \cdot \det (Is - A + LC)$$

Separation principle. The closed-loop eigenvalues are:

$$\{\text{controller poles (roots of } \det(Is - A + BK))\}$$
$$\cup \{\text{observer poles (roots of } \det(Is - A + LC))\}$$

— this holds only for linear systems!!