Dynamic Output Feedback

$$\begin{pmatrix} \dot{x} \\ \dot{\widehat{x}} \end{pmatrix} = \begin{pmatrix} A & -BK \\ LC & A - LC - BK \end{pmatrix} \begin{pmatrix} x \\ \widehat{x} \end{pmatrix}$$

Let us transform to new coordinates:

$$\begin{pmatrix} x \\ \widehat{x} \end{pmatrix} \longmapsto \begin{pmatrix} x \\ e \end{pmatrix} = \begin{pmatrix} x \\ x - \widehat{x} \end{pmatrix} = \underbrace{\begin{pmatrix} I & 0 \\ I & -I \end{pmatrix}}_{T} \begin{pmatrix} x \\ \widehat{x} \end{pmatrix}$$

Two key observations:

- T is invertible, so the new representation is equivalent to the old one
- in the new coordinates, we have

$$\begin{aligned} \dot{x} &= Ax - BK\widehat{x} \\ &= (A - BK)x + BK(x - \widehat{x}) \\ &= (A - BK)x + BKe \\ \dot{e} &= (A - LC)e \end{aligned}$$