Observer in the Presence of Control Input

▶ Let's see what goes wrong when we use the old approach:

$$\dot{\widehat{x}} = (A - LC)\widehat{x} + Ly$$

▶ For the estimation error $e = x - \widehat{x}$, we have

$$
\begin{aligned}
\dot{e} &= \dot{x} - \dot{\widehat{x}} \\
&= Ax + Bu - [(A - LC)\widehat{x} + LCx] \\
&= (A - LC)e + Bu \qquad \text{– not good}
\end{aligned}
$$

▶ Idea: since u is a signal we can access, let's use it as an input to the observer to cancel the Bu term from $\dot{x}$.

▶ Modified observer:

$$
\begin{aligned}
\dot{\widehat{x}} &= (A - LC)\widehat{x} + Ly + Bu \\
\dot{e} &= \dot{x} - \dot{\widehat{x}} \\
&= Ax + Bu - [(A - LC)\widehat{x} + LCx + Bu] \\
&= (A - LC)e \qquad \text{regardless of } u
\end{aligned}
$$