Observability

Consider a single-output system ($y \in \mathbb{R}$):

$$\dot{x} = Ax + Bu, \qquad y = Cx \qquad\qquad x \in \mathbb{R}^n$$

The Observability Matrix is defined as

$$\mathcal{O}(A, C) = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix}$$

— recall that C is $1 \times n$ and A is $n \times n$, so $\mathcal{O}(A, C)$ is $n \times n$;
— the observability matrix only involves A and C, not B

We say that the above system is observable if its observability matrix $\mathcal{O}(A, C)$ is *invertible*.

(This definition is only true for the single-output case; the multiple-output case involves the *rank* of $\mathcal{O}(A, C)$.)