Observer Pole Placement, O/C Duality Version

Given an observable pair (A, C):

1. For $F = A^T$, $G = C^T$, consider the system $\dot{x} = Fx + Gu$ (this system is controllable).
2. Use our earlier procedure to find K, such that

$$F - GK = A^T - C^T K$$

has desired eigenvalues.

3. Then

$$\text{eig}(A^T - C^T K) = \text{eig}(A^T - C^T K)^T = \text{eig}(A - K^T C),$$

so $L = K^T$ is the desired output injection matrix.

Final answer: use the observer

$$\begin{aligned} \dot{\widehat{x}} &= (A - LC)\widehat{x} + Ly \\ &= (A - K^T C)\widehat{x} + K^T y. \end{aligned}$$