

## Controllability–Observability Duality

**Claim:** The system

$$\dot{x} = Ax, \quad y = Cx$$

is observable if and only if the system

$$\dot{x} = A^T x + C^T u$$

is controllable.

**Proof:**  $\mathcal{C}(A^T, C^T) = [C^T \mid A^T C^T \mid \dots \mid (A^T)^{n-1} C^T]$

$$= \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix}^T = [\mathcal{O}(A, C)]^T$$

Thus,  $\mathcal{O}(A, C)$  is nonsingular if and only if  $\mathcal{C}(A^T, C^T)$  is.