Linear ODEs: A Digression

$$\dot{\bar{v}} = TFT^{-1}\bar{v} = \begin{pmatrix} \lambda_1 & & & \\ & \lambda_2 & & \\ & & \ddots & \\ & & & \lambda_n \end{pmatrix} \bar{v}, \qquad (\lambda_1, \ldots, \lambda_n) = \mathrm{eig}(F)$$

$$\Updownarrow$$

$$\dot{\bar{v}}_i = \lambda_i \bar{v}_i, \qquad i = 1, 2, \ldots, n$$

This system of n 1st-order ODEs has the solution

$$\bar{v}_i(t) = \bar{v}_i(0)e^{\lambda_i t}, \qquad i = 1, 2, \ldots, n$$

If all λ_i's have negative real parts, then

$$\begin{aligned} \|v(t)\|^2 &= v(t)^T v(t) = \bar{v}(t)^T \bar{v}(t) \\ &\leq Ce^{-2\sigma_{\min} t}, \qquad \text{where } \sigma_{\min} = \min_{1 \leq i \leq n} |\mathrm{Re}(\lambda_i)| \end{aligned}$$