Linear ODEs and Eigenvalues: A Digression

$$\dot{v} = Fv, \qquad v \in \mathbb{R}^n,\ F \in \mathbb{R}^{n \times n}$$

Let $\lambda_1, \ldots, \lambda_n$ be eigenvalues of F, i.e., roots of $\det(Is - F) = 0$.

Then there exists a matrix $T \in \mathbb{R}^{n \times n}$, such that $T^{-1} = T^T$ and

$$F = T^{-1} \begin{pmatrix} \lambda_1 & & & \\ & \lambda_2 & & \\ & & \ddots & \\ & & & \lambda_n \end{pmatrix} T$$

Consider the change of coordinates $\bar{v} = Tv$. Then

$$\dot{\bar{v}} = TFT^{-1}\bar{v} = \begin{pmatrix} \lambda_1 & & & \\ & \lambda_2 & & \\ & & \ddots & \\ & & & \lambda_n \end{pmatrix} \bar{v}$$