Coordinate Transformations and Observability

Just like controllability, observability is preserved under invertible coordinate transformations:

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned} \qquad \xrightarrow{T} \qquad \begin{aligned} \dot{\bar{x}} &= \bar{A}\bar{x} + \bar{B}u \\ y &= \bar{C}\bar{x} \end{aligned}$$

where $\bar{A} = TAT^{-1}, \qquad \bar{B} = TB, \qquad \bar{C} = CT^{-1}$

If the original system is observable, then

$$T \underbrace{\left[\mathcal{O}(A,C)\right]^{-1}}_{\text{old}} = \underbrace{\left[\mathcal{O}(\bar{A},\bar{C})\right]^{-1}}_{\text{new}}$$

$$\Updownarrow$$

$$T = \underbrace{\left[\mathcal{O}(\bar{A},\bar{C})\right]^{-1}}_{\text{new}} \underbrace{\left[\mathcal{O}(A,C)\right]}_{\text{old}}$$