Coordinate Transformations and Observability

Just like controllability, observability is preserved under invertible coordinate transformations.

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned} \qquad \xrightarrow{\quad T \quad} \qquad \begin{aligned} \dot{\bar{x}} &= \bar{A}\bar{x} + \bar{B}u \\ y &= \bar{C}\bar{x} \end{aligned}$$

where $\bar{A} = TAT^{-1}, \qquad \bar{B} = TB, \qquad \bar{C} = CT^{-1}$

$$\begin{aligned} \mathcal{O}(\bar{A}, \bar{C}) &= \begin{pmatrix} \bar{C} \\ \bar{C}\bar{A} \\ \vdots \\ \bar{C}\bar{A}^{n-1} \end{pmatrix} = \begin{pmatrix} CT^{-1} \\ CT^{-1}TAT^{-1} \\ \vdots \\ CT^{-1}TA^{n-1}T^{-1} \end{pmatrix} \\ &= \begin{pmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{pmatrix} T^{-1} = \mathcal{O}(A, C)T^{-1} \end{aligned}$$