Example: Computing $\mathcal{O}(A, C)$

$$\text{Let} \quad A = \begin{pmatrix} 0 & -6 \\ 1 & -5 \end{pmatrix}, \quad C = \begin{pmatrix} 0 & 1 \end{pmatrix}$$

Here, $n = 2$, $C \in \mathbb{R}^{1 \times 2}$, $A \in \mathbb{R}^{2 \times 2} \implies \mathcal{O}(A, C) \in \mathbb{R}^{2 \times 2}$.

$$\mathcal{O}(A, C) = \begin{bmatrix} C \\ CA \end{bmatrix}$$

where $CA = \begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & -6 \\ 1 & -5 \end{pmatrix} = \begin{pmatrix} 1 & -5 \end{pmatrix}$

$$\therefore \ \mathcal{O}(A, C) = \begin{pmatrix} 0 & 1 \\ 1 & -5 \end{pmatrix}$$

$$\det \mathcal{O}(A, C) = -1 \quad \implies \quad \text{the system is observable}$$

— recall: this system is in Observer Canonical Form (OCF) ...