Coordinate Transformations and State-Space Models

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned} \quad \xrightarrow{T} \quad \begin{aligned} \dot{\bar{x}} &= \bar{A}\bar{x} + \bar{B}u \\ y &= \bar{C}\bar{x} \end{aligned}$$

where $\bar{A} = TAT^{-1}, \qquad \bar{B} = TB, \qquad \bar{C} = CT^{-1}$

- The transfer function does not change.
- The controllability matrix is transformed:

$$\mathcal{C}(\bar{A}, \bar{B}) = T\mathcal{C}(A, B).$$

- The transformed system is controllable if and only if the original one is.
- If the original system is controllable, then

$$T = \mathcal{C}(\bar{A}, \bar{B}) \left[\mathcal{C}(A, B)\right]^{-1}.$$

This gives us a way of systematically passing to CCF.