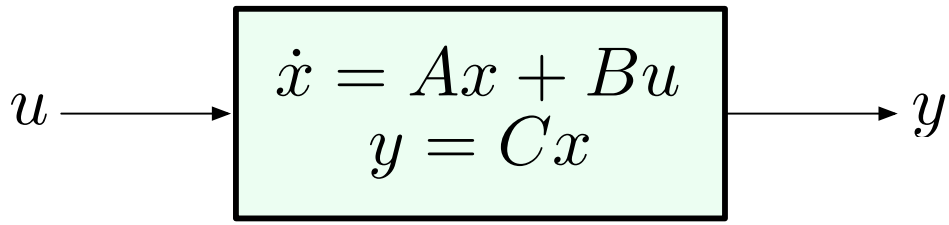
Goal: Pole Placement by State Feedback

Consider a single-input system in state-space form:



Today, our goal is to establish the following fact:

If the above system is *controllable*, then we can assign arbitrary closed-loop poles by means of a state feedback law

$$u = -Kx = -\begin{pmatrix} k_1 & k_2 & \dots & k_n \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}$$

$$= -(k_1 x_1 + \ldots + k_n x_n),$$

where K is a $1 \times n$ matrix of feedback gains.