

Example

Step 2: find $u = -\bar{K}\bar{x}$ to place closed-loop poles at $-10 \pm j$.

Desired characteristic polynomial:

$$(s + 10 + j)(s + 10 - j) = (s + 10)^2 + 1 = s^2 + 20s + 101$$

Thus, the closed-loop system matrix should be

$$\bar{A} - \bar{B}\bar{K} = \begin{pmatrix} 0 & 1 \\ -101 & -20 \end{pmatrix}$$

On the other hand, we know

$$\bar{A} - \bar{B}\bar{K} = \begin{pmatrix} 0 & 1 \\ -(15 + \bar{k}_1) & -(8 + \bar{k}_2) \end{pmatrix} \implies \bar{k}_1 = 86, \bar{k}_2 = 12$$

This gives the control law

$$u = -\bar{K}\bar{x} = - \begin{pmatrix} 86 & 12 \end{pmatrix} \begin{pmatrix} \bar{x}_1 \\ \bar{x}_2 \end{pmatrix}$$