

Proof of the Claim

A nice way is via Laplace transforms:

$$\dot{x} = Ax + Bu$$

$$A = \begin{pmatrix} 0 & 1 & 0 & \dots & 0 & 0 \\ 0 & 0 & 1 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 0 & 1 \\ -a_n & -a_{n-1} & -a_{n-2} & \dots & -a_2 & -a_1 \end{pmatrix}, \quad B = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix}$$

Represent this as a system of ODEs:

$$\dot{x}_1 = x_2$$

$$X_2 = sX_1$$

$$\dot{x}_2 = x_3$$

$$X_3 = sX_2 = s^2X_1$$

$$\vdots$$
$$\vdots$$

$$\dot{x}_n = -\sum_{i=1}^n a_{n-i+1}x_i + u$$

$$\underbrace{(s^n + a_1s^{n-1} + \dots + a_n)}_{\text{char. poly.}} X_1 = U$$