Beware of Pole-Zero Cancellations!!

We can look at this from another angle: consider the t.f.

$$G(s) = \frac{1}{s+3}$$

We can realize it using a one-dimensional controllable state-space model

$$\dot{x}_1 = -3x_1 + u, \quad y = x_1$$

or a noncontrollable two-dimensional state-space model

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} 0 & -6 \\ 1 & -5 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 2 \\ 1 \end{pmatrix} u, \qquad y = \begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

— certainly not the best way to realize a simple t.f.!

Thus, even the *state dimension* of a realization of a given t.f. is not unique!!