## Beware of Pole-Zero Cancellations!

The OCF realization of the transfer function

$$G(s) = \frac{s - z}{s^2 + 5s + 6}$$

is not controllable when z = -2 or -3, even though the CCF is always controllable.

Let's examine G(s) when z = -2:

$$G(s) = \frac{s-z}{s^2+5s+6}\Big|_{z=-2} = \frac{s+2}{(s+2)(s+3)} = \frac{1}{s+3}$$

— pole-zero cancellation!

For z = -2, G(s) is a first-order transfer function, which can always be realized by this 1st-order controllable model:

$$\dot{x}_1 = -3x_1 + u, \ y = x_1 \quad \longrightarrow \quad G(s) = \frac{1}{s+3}$$