OCF with Arbitrary Zeros

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \underbrace{\begin{pmatrix} 0 & -6 \\ 1 & -5 \end{pmatrix}}_{\bar{A}=A^T} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \underbrace{\begin{pmatrix} -z \\ 1 \end{pmatrix}}_{\bar{B}=C^T} u, \qquad y = \underbrace{\begin{pmatrix} 0 & 1 \end{pmatrix}}_{\bar{C}=B^T} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

Let's find the controllability matrix:

$$\mathcal{C}(\bar{A}, \bar{B}) = \left[\bar{B} \mid \bar{A}\bar{B}\right] \qquad \bar{A}\bar{B} = \begin{pmatrix} 0 & -6 \\ 1 & -5 \end{pmatrix} \begin{pmatrix} -z \\ 1 \end{pmatrix} = \begin{pmatrix} -6 \\ -z-5 \end{pmatrix}$$

$$\therefore \mathcal{C}(\bar{A}, \bar{B}) = \begin{pmatrix} -z & -6 \\ 1 & -z-5 \end{pmatrix}$$

$$\det \mathcal{C} = z(z+5) + 6 = z^2 + 5z + 6 = 0 \quad \text{for } z = -2 \text{ or } z = -3$$

The OCF realization of the transfer function $G(s) = \dfrac{s-z}{s^2+5s+6}$ is not controllable when $z = -2$ or -3, even though the CCF is always controllable.