Coordinate Transformations and State-Space Models

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned} \qquad \xrightarrow{\quad T \quad} \qquad \begin{aligned} \dot{\bar{x}} &= \bar{A}\bar{x} + \bar{B}u \\ y &= \bar{C}\bar{x} \end{aligned}$$

where $\bar{A} = TAT^{-1}, \qquad \bar{B} = TB, \qquad \bar{C} = CT^{-1}$

Note: The *controllability matrix* does change:

$$\underbrace{\mathcal{C}(\bar{A}, \bar{B})}_{\text{new}} = \underbrace{T}_{\substack{\text{coord.}\\ \text{trans.}}} \underbrace{\mathcal{C}(A, B)}_{\text{old}}$$

$$\Updownarrow$$

$$T = \mathcal{C}(\bar{A}, \bar{B}) \left[\mathcal{C}(A, B)\right]^{-1}$$

This is a recipe for going from one *controllable* realization of a given t.f. to another.

CCF is the most convenient controllable realization of a given t.f., so we want to *convert a given controllable system to CCF* (useful for control design).