

Coordinate Transformations and State-Space Models

$$\begin{array}{ccc} \dot{x} = Ax + Bu & \xrightarrow{T} & \dot{\bar{x}} = \bar{A}\bar{x} + \bar{B}u \\ y = Cx & & y = \bar{C}\bar{x} \end{array}$$

$$\text{where } \bar{A} = TAT^{-1}, \quad \bar{B} = TB, \quad \bar{C} = CT^{-1}$$

The transfer function doesn't change.

In fact:

- ▶ open-loop poles don't change
- ▶ characteristic polynomial doesn't change:

$$\begin{aligned} \det(Is - \bar{A}) &= \det(Is - TAT^{-1}) \\ &= \det [T(Is - A)^{-1}T^{-1}] \\ &= \det T \cdot \det(Is - A)^{-1} \cdot \det T^{-1} \\ &= \det(Is - A)^{-1} \end{aligned}$$