Coordinate Transformations and State-Space Models

$$\begin{aligned} \dot{x} &= Ax + Bu \qquad \xrightarrow{\quad T \quad} \qquad \dot{\bar{x}} = \bar{A}\bar{x} + \bar{B}u \\ y &= Cx \qquad\qquad\qquad\qquad\qquad\quad y = \bar{C}\bar{x} \end{aligned}$$

where $\bar{A} = TAT^{-1}, \qquad \bar{B} = TB, \qquad \bar{C} = CT^{-1}$

Claim: The transfer function doesn't change.

Proof:

$$\begin{aligned} \bar{G}(s) &= \bar{C}(Is - \bar{A})^{-1}\bar{B} \\ &= (CT^{-1})\left(Is - TAT^{-1}\right)^{-1}(TB) \\ &= CT^{-1}\left(TIT^{-1}s - TAT^{-1}\right)^{-1}TB \\ &= CT^{-1}\left[T\left(Is - A\right)T^{-1}\right]^{-1}TB \\ &= C\underbrace{T^{-1}T}_{I}(Is - A)^{-1}\underbrace{T^{-1}T}_{I}B \\ &= C\left(Is - A\right)^{-1}B \equiv G(s) \end{aligned}$$