Coordinate Transformations and State-Space Models

Consider a state-space model

$$\dot{x} = Ax + Bu$$
$$y = Cx$$

and a change of coordinates $\bar{x} = Tx$ (T invertible).

What does the system look like in the new coordinates?

$$\begin{aligned}
\dot{\bar{x}} &= T\dot{x} = T\dot{x} && \text{(linearity of derivative)} \\
&= T(Ax + Bu) \\
&= T(AT^{-1}\bar{x} + Bu) && (x = T^{-1}\bar{x}) \\
&= \underbrace{TAT^{-1}}_{\bar{A}}\bar{x} + \underbrace{TB}_{\bar{B}}u \\
y &= Cx \\
&= \underbrace{CT^{-1}}_{\bar{C}}\bar{x}
\end{aligned}$$