Coordinate Transformations

For example,

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \longmapsto \begin{pmatrix} \bar{x}_1 \\ \bar{x}_2 \end{pmatrix} = \begin{pmatrix} x_1 + x_2 \\ x_1 - x_2 \end{pmatrix}$$

This can be represented as

$$\bar{x} = Tx, \qquad \text{where } T = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

The transformation is invertible: $\det T = -2$, and

$$T^{-1} = \frac{1}{\det T} \begin{pmatrix} -1 & -1 \\ -1 & 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{pmatrix}$$

Or we can see this directly:

$$\bar{x}_1 + \bar{x}_2 = 2x_1; \quad \bar{x}_1 - \bar{x}_2 = 2x_2$$