From State-Space to Transfer Function

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \end{aligned} \qquad \begin{aligned} Y(s) &= G(s)U(s) \\ &= \left[C(Is - A)^{-1}B + D\right]U(s) \end{aligned}$$

Important!!

- $G(s)$ is *undefined* when the $n \times n$ matrix $Is - A$ is *singular* (or noninvertible), i.e., precisely when $\det(Is - A) = 0$
- since A is $n \times n$, $\det(Is - A)$ is a *polynomial* of degree n (the characteristic polynomial of A):

$$\det(Is - A) = \det \begin{pmatrix} s - a_{11} & -a_{12} & \dots & -a_{1n} \\ -a_{21} & s - a_{22} & \dots & -a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ -a_{n1} & -a_{n2} & \dots & s - a_{nn} \end{pmatrix},$$

  and its roots are the eigenvalues of A
- G is (open-loop) stable if all eigenvalues of A lie in LHP.