

## From State-Space to Transfer Function

$$y_\ell = \sum_{j=1}^n c_{\ell j} x_j + \sum_{k=1}^m d_{\ell k} u_k$$

$\downarrow \mathcal{L}$

$$Y_\ell(s) = \sum_{j=1}^n c_{\ell j} X_j(s) + \sum_{k=1}^m d_{\ell k} U_k(s), \quad \ell = 1, \dots, p$$

Write down in matrix-vector form:

$$\begin{aligned} Y(s) &= CX(s) + DU(s) \\ &= C \left[ (Is - A)^{-1} x(0) + (Is - A)^{-1} BU(s) \right] + DU(s) \\ &= C(Is - A)^{-1} x(0) + [C(Is - A)^{-1} B + D] U(s) \end{aligned}$$

To find the input-output t.f., set the IC to 0:

$$Y(s) = G(s)U(s), \quad \text{where } G(s) = C(Is - A)^{-1}B + D$$