

## Example: Computing $\mathcal{C}(A, B)$

Let's get back to our old friend:

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \underbrace{\begin{pmatrix} 0 & 1 \\ -6 & -5 \end{pmatrix}}_A \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \underbrace{\begin{pmatrix} 0 \\ 1 \end{pmatrix}}_B u, \quad y = \underbrace{\begin{pmatrix} 1 & 1 \end{pmatrix}}_C \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

Here,  $x \in \mathbb{R}^2 \implies A \in \mathbb{R}^{2 \times 2} \implies \mathcal{C}(A, B) \in \mathbb{R}^{2 \times 2}$

$$\begin{aligned} \mathcal{C}(A, B) &= [B \mid AB] & AB &= \begin{pmatrix} 0 & 1 \\ -6 & -5 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ -5 \end{pmatrix} \\ \implies \mathcal{C}(A, B) &= \begin{pmatrix} 0 & 1 \\ 1 & -5 \end{pmatrix} \end{aligned}$$

Is this system controllable?

$$\det \mathcal{C} = -1 \neq 0 \quad \implies \quad \text{system is controllable}$$