Controllability Matrix

Consider a single-input system ($u \in \mathbb{R}$):

$$\dot{x} = Ax + Bu, \qquad y = Cx \qquad\qquad x \in \mathbb{R}^n$$

The Controllability Matrix is defined as

$$\mathcal{C}(A, B) = \left[B \mid AB \mid A^2B \mid \ldots \mid A^{n-1}B \right]$$

— recall that A is $n \times n$ and B is $n \times 1$, so $\mathcal{C}(A, B)$ is $n \times n$;
— the controllability matrix only involves A and B, not C

We say that the above system is controllable if its controllability matrix $\mathcal{C}(A, B)$ is *invertible*.

(This definition is only true for the single-input case; the multiple-input case involves the *rank* of $\mathcal{C}(A, B)$.)