State-Space Realizations of Transfer Functions

Claim: The state-space model

$$\dot{x} = \bar{A}x + \bar{B}u, \qquad y = \bar{C}x$$

with

$$\bar{A} = A^T, \quad \bar{B} = C^T, \quad \bar{C} = B^T$$

has the same transfer function as the original model with (A, B, C).

Proof:

$$\begin{aligned}
\bar{C}(Is - \bar{A})^{-1}\bar{B} &= B^T \left(Is - A^T\right)^{-1} C^T \\
&= B^T \left[(Is - A)^T\right]^{-1} C^T \\
&= B^T \left[(Is - A)^{-1}\right]^T C^T \\
&= \left[C(Is - A)^{-1}B\right]^T \\
&= C(Is - A)^{-1}B
\end{aligned}$$