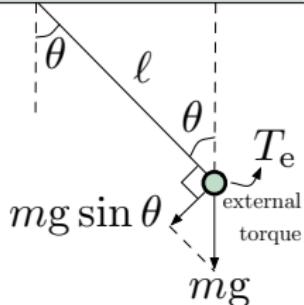# Example 3: Pendulum

Newton's 2nd law (rotational motion):

$$\underbrace{T}_{\substack{\text{total} \\ \text{torque}}} = \underbrace{J}_{\substack{\text{moment} \\ \text{of inertia}}} \underbrace{\alpha}_{\substack{\text{angular} \\ \text{acceleration}}}$$

$$= \text{pendulum torque} + \text{external torque}$$

$$\text{pendulum torque} = \underbrace{-mg\sin\theta}_{\text{force}} \cdot \underbrace{\ell}_{\text{lever arm}}$$

$$\text{moment of inertia } J = m\ell^2$$

$$-mg\ell\sin\theta + T_e = m\ell^2\ddot{\theta}$$

$$\ddot{\theta} = -\frac{g}{\ell}\sin\theta + \frac{1}{m\ell^2}T_e \qquad \text{(nonlinear equation)}$$