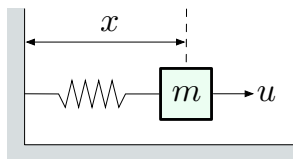


Example 1: Mass-Spring System



Newton's second law (translational motion):

$$\underbrace{F}_{\text{total force}} = ma = \text{spring force} + \text{friction} + \text{external force}$$

$$\text{spring force} = -kx \quad (\text{Hooke's law})$$

$$\text{friction force} = -\rho\dot{x} \quad (\text{Stokes' law} \text{ — linear drag, only an approximation!!})$$

$$m\ddot{x} = -kx - \rho\dot{x} + u$$

Move x, \dot{x}, \ddot{x} to the LHS, u to the RHS:

$$\ddot{x} + \frac{\rho}{m}\dot{x} + \frac{k}{m}x = \frac{u}{m} \quad \text{2nd-order linear ODE}$$