General Linearization Procedure

- Pass to shifted variables $\underline{x} = x - x_0$, $\underline{u} = u - u_0$

$$\begin{aligned} \dot{\underline{x}} &= \dot{x} && (x_0 \text{ does not depend on } t) \\ &= f(x,u) \\ &= \underline{f}(\underline{x},\underline{u}) \end{aligned}$$

— equivalent to original system

- The transformed system is in equilibrium at $(0,0)$:

$$\underline{f}(0,0) = f(x_0,u_0) = 0$$

- Now linearize:

$$\dot{\underline{x}} = A\underline{x} + B\underline{u}, \qquad \text{where } A_{ij} = \left.\frac{\partial f_i}{\partial x_j}\right|_{\substack{x=x_0 \\ u=u_0}}, \; B_{ik} = \left.\frac{\partial f_i}{\partial u_k}\right|_{\substack{x=x_0 \\ u=u_0}}$$