# Example 3: Pendulum, Revisited

Original nonlinear state-space model:

$$\dot{\theta}_1 = f_1(\theta_1, \theta_2, T_\mathrm{e}) = \theta_2 \qquad \text{— already linear}$$

$$\dot{\theta}_2 = f_2(\theta_1, \theta_2, T_\mathrm{e}) = -\frac{\mathrm{g}}{\ell}\sin\theta_1 + \frac{1}{m\ell^2}T_\mathrm{e}$$

Linear approx. of $f_2$ around equilibrium $(\theta_1, \theta_2, T_\mathrm{e}) = (0, 0, 0)$:

$$\frac{\partial f_2}{\partial \theta_1} = -\frac{\mathrm{g}}{\ell}\cos\theta_1 \qquad \frac{\partial f_2}{\partial \theta_2} = 0 \qquad \frac{\partial f_2}{\partial T_\mathrm{e}} = \frac{1}{m\ell^2}$$

$$\left.\frac{\partial f_2}{\partial \theta_1}\right|_0 = -\frac{\mathrm{g}}{\ell} \qquad \left.\frac{\partial f_2}{\partial \theta_2}\right|_0 = 0 \qquad \left.\frac{\partial f_2}{\partial T_\mathrm{e}}\right|_0 = \frac{1}{m\ell^2}$$

Linearized state-space model of the pendulum:

$$\dot{\theta}_1 = \theta_2$$

$$\dot{\theta}_2 = -\frac{\mathrm{g}}{\ell}\theta_1 + \frac{1}{m\ell^2}T_\mathrm{e}$$

valid for *small* deviations from equ.