Linearization

Linear approx. around $(x, u) = (0, 0)$ to all components of f:

$$\dot{x}_1 = f_1(x, u), \qquad \ldots, \qquad \dot{x}_n = f_n(x, u)$$

For each $i = 1, \ldots, n$,

$$f_i(x, u) = \underbrace{f_i(0, 0)}_{=0} + \frac{\partial f_i}{\partial x_1}(0, 0)x_1 + \ldots + \frac{\partial f_i}{\partial x_n}(0, 0)x_n + \frac{\partial f_i}{\partial u_1}(0, 0)u_1 + \ldots + \frac{\partial f_i}{\partial u_m}(0, 0)u_m$$

Linearized state-space model:

$$\dot{x} = Ax + Bu, \qquad \text{where } A_{ij} = \left.\frac{\partial f_i}{\partial x_j}\right|_{\substack{x=0 \\ u=0}}, \; B_{ik} = \left.\frac{\partial f_i}{\partial u_k}\right|_{\substack{x=0 \\ u=0}}$$

Important: since we have ignored the higher-order terms, this linear system is only an *approximation* that holds only for *small deviations* from equilibrium.