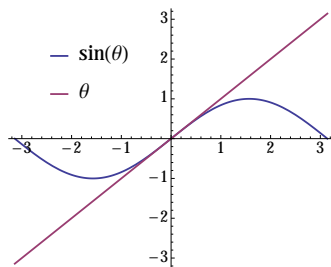


Example 3: Pendulum

$$\ddot{\theta} = -\frac{g}{l} \sin \theta + \frac{1}{m\ell^2} T_e \quad (\text{nonlinear equation})$$

For *small* θ , use the approximation $\sin \theta \approx \theta$



$$\ddot{\theta} = -\frac{g}{l} \theta + \frac{1}{m\ell^2} T_e$$

State-space form: $\theta_1 = \theta$, $\theta_2 = \dot{\theta}$

$$\dot{\theta}_2 = -\frac{g}{l} \theta + \frac{1}{m\ell^2} T_e = -\frac{g}{l} \theta_1 + \frac{1}{m\ell^2} T_e$$

$$\begin{pmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -\frac{g}{l} & 0 \end{pmatrix} \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix} + \begin{pmatrix} 0 \\ \frac{1}{m\ell^2} \end{pmatrix} T_e$$