

The Argument Principle

These special cases all lead to the following general result:

The Argument Principle. Let C be a closed, clockwise \circlearrowright oriented contour not passing through any zeros or poles* of $H(s)$. Let $H(C)$ be the image of C under the map $s \mapsto H(s)$:

$$H(C) = \{H(s) : s \in C\}.$$

Then:

$$\begin{aligned} & \#(\text{clockwise encirclements } \circlearrowright \text{ of } 0 \text{ by } H(C)) \\ &= \#(\text{zeros of } H(s) \text{ inside } C) - \#(\text{poles of } H(s) \text{ inside } C). \end{aligned}$$

More succinctly,

$$N = Z - P$$

* will see the reason for this later ...