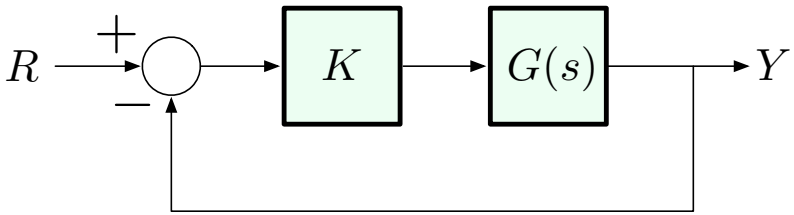
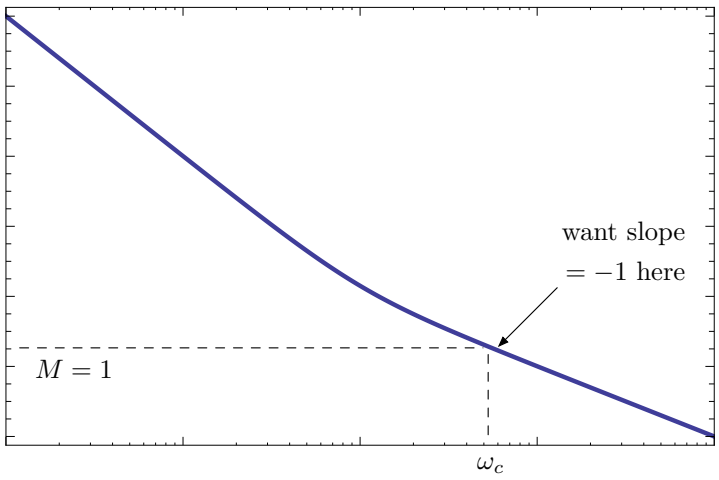
Gain-Phase Relationship & Bandwidth



$$\begin{cases} |KG(j\omega_c)| = 1 \\ \angle G(j\omega_c) = -90^\circ \end{cases} \Rightarrow KG(j\omega_c) = -j$$

M-plot for *open-loop* t.f. KG:



Note: $|KG(j\omega)| \to \infty$ as $\omega \to 0$

Closed-loop t.f.:

$$T(j\omega_c) = \frac{KG(j\omega_c)}{1 + KG(j\omega_c)} = \frac{-j}{1-j}$$

$$|T(j\omega_c)| = \left|\frac{-j}{1-j}\right| = \frac{1}{\sqrt{2}}$$

$$|T(0)| = \lim_{\omega \to 0} \frac{|KG(j\omega)|}{|1 + KG(j\omega)|} = 1$$

$$\Longrightarrow \omega_c = \omega_{\text{BW}} \text{ (bandwidth)}$$

- If PM $= 90^\circ$, then $\omega_c = \omega_{\text{BW}}$
- If $PM < 90^\circ$, then $\omega_c \le \omega_{\text{BW}} \le 2\omega_c$ (see FPE)