

## Phase Margin for 2nd-Order System

$$G(j\omega) = \frac{\omega_n^2}{(j\omega)^2 + 2\zeta\omega_n j\omega} = \frac{\omega_n}{2\zeta j\omega \left( \frac{j\omega}{2\zeta\omega_n} + 1 \right)}$$

$$\text{PM}\Big|_{K=1} = \tan^{-1} \left( \frac{2\zeta}{\sqrt{4\zeta^4 + 1} - 2\zeta^2} \right) \approx 100 \cdot \zeta$$

### Conclusions:

larger PM  $\iff$  better damping  
(open-loop quantity) (closed-loop characteristic)

Thus, the overshoot  $M_p = \exp\left(-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}\right)$  and resonant peak  $M_r = \frac{1}{2\zeta\sqrt{1-\zeta^2}} - 1$  are both related to PM through  $\zeta!!$