

Approximate PI via Lag Compensation

$$G_c(s) = K \frac{s+z}{s+p}, \quad p < z \qquad G_p(s) = \frac{1}{s-1}$$

How good is this controller?

Tracking a constant reference: assuming closed-loop stability, the FVT gives

$$e(\infty) = \frac{1}{1 + G_c(s)G_p(s)} \Big|_{s=0} = \frac{1}{1 + K \frac{s+z}{(s+p)(s-1)}} \Big|_{s=0} = \frac{1}{1 - \frac{Kz}{p}}$$

Check for stability: no RHP poles for $\frac{1}{1 + G_c(s)G_p(s)}$

$$(s+p)(s-1) + K(s+z) = 0$$

$$s^2 + (K+p-1)s + Kz - p = 0$$

Conditions for stability: $K > 1 - p$, $Kz > p$