

The Frequency-Response Design Method

$$\sin(\omega t) \longrightarrow \boxed{G(s)} \longrightarrow M \sin(\omega t + \phi)$$

where $M = M(\omega) = |G(j\omega)|$ and $\phi = \phi(\omega) = \angle G(j\omega)$

Let's apply this formula to our prototype 2nd-order system:

$$\begin{aligned} G(s) &= \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \\ M(\omega) = |G(j\omega)| &= \left| \frac{\omega_n^2}{-\omega^2 + 2j\zeta\omega_n\omega + \omega_n^2} \right| \\ &= \left| \frac{1}{1 - \left(\frac{\omega}{\omega_n}\right)^2 + 2\zeta\frac{\omega}{\omega_n}j} \right| \\ &= \frac{1}{\sqrt{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + 4\zeta^2\left(\frac{\omega}{\omega_n}\right)^2}} \end{aligned}$$